PHYC 581: High Energy Astrophysics
Fall 2014

Lecture 5: 09/03/2014

Particle Acceleration:

It is important to understand how particles in high-energy sources accelerate to Lorentz factors required to emit X-ray and γ-ray photons. Although large structures like molecular clouds or galaxy clusters can contribute significantly to the ambient high-energy intensity, the overwhelming majority of energetic particles are produced in and around compact objects (such as white dwarfs, neutron stars, and black holes).

Particle acceleration in and around compact objects is due to the strong fields induced by these objects. Here we discuss acceleration of particles in gravitational and electromagnetic fields. The main purpose will be to

Compare the optimal efficiency of various processes in transferring energy from the fields to the particles.

Gravitational Field:

A particle falling from infinity onto an object of mass M and radius R can acquire a maximum velocity given by:

$$v_{max} = \left(\frac{2GM}{R}\right)^{\frac{1}{2}}$$

For a white dwarf ($M_{WD} \sim M_\odot$ and $R_{WD} \sim 10^4$ km), we have:

$$v_{max}(WD) = \left(\frac{2GM_{WD}}{R_{WD}}\right)^{1/2} \sim 7.4 \times 10^8 \ cm/s \Rightarrow \gamma_{max}(WD) \sim 1.0003$$

It is clear that the gravitational field of a white dwarf alone cannot produce relativistic particles.

Next, we consider a neutron star ($M_{NS} \sim M_\odot$, $R_{NS} \sim 10$ km):

$$v_{max}(NS) = \left(\frac{2GM_{NS}}{R_{NS}}\right)^{1/2} \sim \frac{c}{2} \Rightarrow \gamma_{max}(NS) \sim 1.19$$

This is definitely more interesting than that for a white dwarf. However, the Lorentz factor is still far too small for a particle like electron or proton to radiate energetic γ-ray photons. For example, to radiate a 30 GeV γ-ray photon, we need $\gamma \sim 30$ (for a proton) and $\gamma \sim 60{,}000$ (for an electron).

It is also important to note that photons produced near a compact object undergo gravitational redshift. The redshift factor, which we discussed last time, is:

$$z = \left(1 - \frac{2GM}{Rc^2}\right)^{-\frac{1}{2}} - 1$$

For a black hole, photons are emitted from regions outside the horizon, i.e., $r \geq R_c \left(= \frac{2GM}{c^2}\right)$. It is seen that $z \to \infty$ as $r \to R_c$. Therefore, even though the maximum speed attainable by matter falling inward is c, the gravitational redshift becomes infinite at the horizon, which more than offsets the increase in γ as $r \to R_s$. Hence, very little high-energy emission is expected from regions near the horizon itself.

Electromagnetic Field:

It is clear from our discussion in above that non-gravitational acceleration schemes must play a role in energizing the particles in many high-energy sources. The most common is the electromagnetic acceleration, via at least two methods of energy transfer. Depending on the field distribution, various mechanisms may contribute to the acceleration.

When the magnetic field $\vec{B}$ is turbulent or random, the principal method is the Fermi acceleration. In this process, disordered bundles of magnetic flux act as mirrors to bounce particles back and forth, increasing their energy with every collision. We will discuss this mechanism in detail later on. For a well-organized field, a more direct acceleration mechanism is based on the idea that a component of the electric field $\vec{E}$ may be generated parallel to $\vec{B}$, where the particle motion is unrestricted. This is the situation that we consider in detail now.

The Lorentz force on a charge q in the presence of the electric and magnetic fields $\vec{E}$ and $\vec{B}$, respectively, is (we use Gaussian units):

$$\vec{F} = q\left(\vec{E} + \frac{\vec{v}}{c} \times \vec{B}\right)$$

It is clear that the magnetic field cannot increase the speed of the charged particle since the force that it produces is perpendicular to $\vec{v}$. Therefore, the electric field is necessary in order to increase v. In the presence of a strong magnetic field, the charge is confined to circular motion about the magnetic field line with a very small Larmor radius. The charge is practically attached to magnetic field lines and can move only in the parallel direction. Hence, in order to increase v, there must exist a component of $\vec{E}$ that is parallel to $\vec{B}$.

To understand how a magnetic field disturbance energizes the charges, let us go through some of the basic ideas of magnetohydrodynamics (MHD). First, consider Maxwell's equations:

$$\vec{\nabla} \cdot \vec{E} = 4\pi \rho_e \qquad \vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{B} = \frac{4\pi}{c} \vec{J} + \frac{1}{c} \frac{\partial \vec{E}}{\partial t}$$

Here ρ_e is the density of electric charge and $\vec{J}$ is the currend density. Additional relations are found by considering a highly conducting medium such as a plasma of a fully inonized gas. This is a typical situation that occurs in the magnetosphere of a pulsa. A pulsar can be considered as a rotating magnetized sphere (thus a rotating magnetic dipole). The induced electric field by this rotation ionizes the gas in the magnetosphere. In a highly conducting medium, we have:

$$\vec{E} + \frac{\vec{v}}{c} \times \vec{B} = 0$$

Here $\vec{v}$ is the velocity field of the plasma. Moreover, the conservation of mass leads to the following equation:

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) = 0$$

ρ is the mass density of the plasma. The equivalent of the Newton's second law in the plasma is:

$$\rho \frac{\partial \vec{v}}{\partial t} + \rho (\vec{v} \cdot \vec{\nabla}) \vec{v} = \rho_e \vec{E} + \frac{1}{c} (\vec{J} \times \vec{B})$$

For an electrically neutral medium $\rho_e = 0$. After using the Maxwell's equations, we find:

$$\rho \frac{\partial \vec{v}}{\partial t} + \rho (\vec{v} \cdot \vec{\nabla}) \vec{v} = -\frac{1}{4\pi} \vec{B} \times (\vec{\nabla} \times \vec{B})$$

This relation holds in the non-relativistic limit when

$$\vec{\nabla} \times \vec{B} - \frac{1}{c} \frac{\partial \vec{E}}{\partial t} \simeq \vec{\nabla} \times \vec{B}.$$

Considering the conservation of mass and the condition for a highly conducting medium, we have two more equations.

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) = 0$$

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times (\vec{v} \times \vec{B})$$

The three equations form a complete set of coupled equations. Suppose now that the field $\vec{B}$ is subject to a disturbance. In a pulsar, this can happen, for example, as a result of a crustal disturbance. The solution to the above equations that describes a propagating plane wave (in the $\hat{z}$ direction) is:

$\vec{B} = B_0 \hat{z} + B_A \exp(ikz - i\omega t) \hat{x}$

$\vec{v} = v_A \exp(ikz - i\omega t) \hat{x}$

$\vec{E} = E_A \exp(ikz - i\omega t) \hat{y}$

This is called the "Alfven wave" (hence the subscript "A").

We notice the presence of an electric field $\vec{E}$! However, it is perpendicular to $\vec{B}$. It can therefore not help accelerate particles along the magnetic field lines (pulsars have very strong magnetic fields up to $\sim 10^{12}$ G).

A parallel component of $\vec{E}$ can arise if the plane wave has a finite extension in the x-y plane. In reality, one expects some structure in the x-y plane due to the pulsar crust.

A solution that satisfies the above equations and takes this into account is;

$$\vec{B} = B_0 \hat{z} + B_A \sin(k_y y) \exp(ikz - i\omega t)\, \hat{z}$$

$$\vec{E} \approx \frac{ic}{\omega}\left[ik B_A \sin(k_y y)\,\hat{y} - k_y B_A \cos(k_y y)\,\hat{z}\right] \exp(ikz - i\omega t)$$

The z component of $\vec{E}$ can now result in acceleration of charged particles. The equation of motion for an electron along the z direction is;

$$\frac{dP_z}{dt} = eE_z \Rightarrow \frac{d}{dt}(\gamma m_e v_z) = eE_z$$

For relativistic motion $v_z \simeq c$. The major contribution to the left-hand side therefore comes from $\frac{d\gamma}{dt}$. We find;

$$\frac{d\gamma}{dt} \simeq \frac{eE_z}{m_e c}$$

This equation is valid when the electron moves freely between two successive collisions with particles in the plasma.

Thus;

$$\gamma_{max} \simeq \frac{eE_z}{m_e c \nu_c}$$

Here ν_c is the collision frequency. In a typical pulsar magnetosphere, where the particle density is $\sim 10^{16} - 10^{26}\ cm^{-3}$, we have $\nu_c \sim 10^{7}\ s^{-1}$. Also, in a typical pulsar, $k_{\parallel} \sim \frac{2\pi}{\lambda_{crust}}$, where the crust scale length is $\lambda_{crust} \sim 10\ cm$. In addition, $\omega \simeq V_A k$, where V_A is the phase velocity of the Alfven wave and $k \sim \frac{2\pi}{R_{NS}}$.

We therefore find that an electron can be accelerated to a Lorentz factor of 10^{10} or more. In practice, several damping factors (such as pair creation) will set in before reaching such high values of γ.